Signal Processing over graphs

The goal of the course is to illustrate basic methodologies for graph signal processing suitable for the very general case of signals defined over non-metric space domains, like for example in gene regulatory networks. Graph-theoretical tools play a fundamental role in this new formulation and then they are deeply covered during the course. Graph models are discussed and analyzed, together with operations over graphs, like partitioning. Convex optimization is then presented and applied to several examples, from resource allocation in communication networks to machine learning. Finally, it is shown how to formulate and solve optimization problems in distributed form, suitable for big data applications, where the amount of data is so big that the data cannot reside on a single machine, but are spread over a high number of machines.

Prerequisites: Basic knowledge of calculus, probability theory and statistics.

Program:

Part I: Recap on signal properties, discrete representations, Fourier transforms, filtering, sampling theory, applications to audio signals and images. Sparse representations, compressive sensing

Part II: Processing over graphs.

- Algebraic graph theory, graph properties, connectivity.
- Graph features: degree centrality, eigenvector centrality, PageRank, betweeness, modularity
- Graph models: random graphs, random geometric graphs, small worlds graphs, scale-free graphs.
- Independence graphs: Markov networks, Bayes networks, Gaussian Markov Random Fields
- Operations on graphs: partitioning.
- Signals defined on graphs.
- Filtering and sampling signals over graphs.
- Prediction of processes over graphs.
- Inference of graph topology from data.

Part III: Recap on Convex optimization: alternating direction method of multipliers, algorithms for sparsity constrained problems-

Part IV: **Examples of application**

- Graph-based methods for machine learning
- Graph topology inference from data (brain, finance, ...)
- Initial concept on. Deep Neural Network Approximation Theory

Evaluation:

- There will be a few homework assignments that involve both theory and programming components. A hard copy of your homework is required. Students are encouraged to use LaTeX to typeset their homeworks. A final project, at the end of the course, will be also assigned. The final evaluation will be based on the homework assignments, the project and an oral exam.

During the course material in form of slides and lecture notes will be provided.